$with(LinearAlgebra):$

$f := (x, y) \rightarrow \ln(x^2 + y^2 - 2 \cdot x \cdot y + 1)$

$$(x, y) \rightarrow \ln(x^2 + y^2 - 2\,x\,y + 1) \tag{1}$$

$g := (x, y) \rightarrow \sqrt{x^2 + y^2}$

$$(x, y) \rightarrow \sqrt{x^2 + y^2} \tag{2}$$

$c := 1$

$$1 \tag{4}$$

$\Lambda := (x, y, \lambda) \rightarrow f(x, y) + \lambda \cdot (g(x, y) - c)$

$$(x, y, \lambda) \rightarrow f(x, y) + \lambda\,(g(x, y) - c) \tag{5}$$

$\frac{d}{d\,x}\,\Lambda(x, y, \lambda)$

$$\frac{2\,x - 2\,y}{x^2 + y^2 - 2\,x\,y + 1} + \frac{\lambda\,x}{\sqrt{x^2 + y^2}} \tag{6}$$

$\frac{d}{d\,y}\,\Lambda(x, y, \lambda)$

$$\frac{2\,y - 2\,x}{x^2 + y^2 - 2\,x\,y + 1} + \frac{\lambda\,y}{\sqrt{x^2 + y^2}} \tag{7}$$

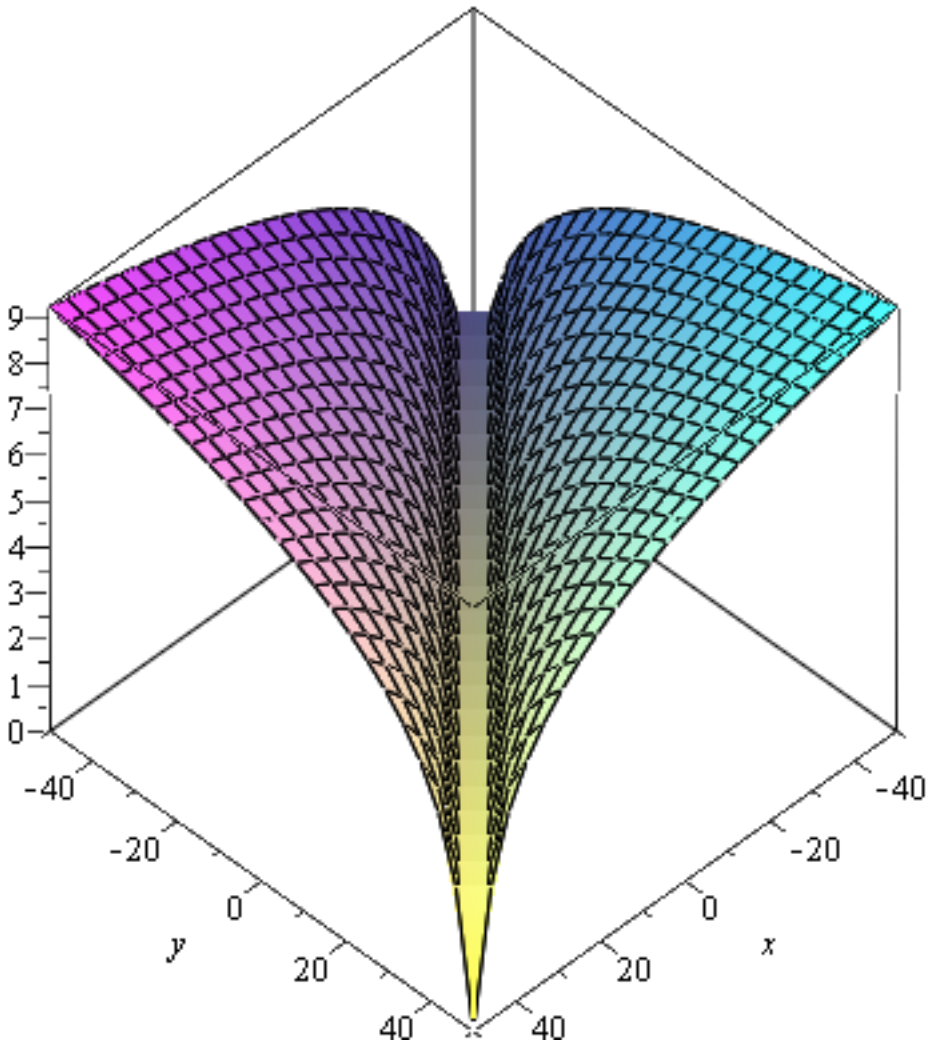
$\frac{d}{d\,\lambda}\,\Lambda(x, y, \lambda)$

$$\sqrt{x^2 + y^2} - 1 \tag{8}$$

$solve\left(\left\{\frac{d}{d\,x}\,\Lambda(x, y, \lambda) = 0, \frac{d}{d\,y}\,\Lambda(x, y, \lambda) = 0, \frac{d}{d\,\lambda}\,\Lambda(x, y, \lambda) = 0\right\}, [x, y, \lambda]\right)$

$$\left[[x = RootOf(2\,\_Z^2 - 1), y = RootOf(2\,\_Z^2 - 1), \lambda = 0], \left[x = -RootOf(2\,\_Z^2 - 1), y = RootOf(2\,\_Z^2 - 1), \lambda = -\frac{4}{3}\right]\right] \tag{9}$$

$plot3d([f(x, y)], x = -50..50, y = -50..50, axes = boxed)$


9
8
7
6
5
4
3
2
1
0
-40
-20
0
y
20
40
-40
-20
0
x
20
40